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INFLUENCE OF LIQUID HEAT CONDUCTION ON MAXIMUM PRESSURE DURING TRANSIENT FILM BOILING FROM A SPHERE TO A SATURATED LIQUID

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Abstract—The maximum pressure excursions which occur in the pure vapor film enveloping a very hot sphere of constant temperature which is suddenly exposed to a pool of saturated and stagnant liquid are analytically evaluated. Heat conduction in the incompressible liquid, arising from saturation temperature's dependence upon pressure, is shown to be important. Generalized predictions for maximum pressure are presented in the form of graphs, and approximative equations are derived.

NOMENCLATURE

A, parameter, $A^2 = P_\infty/R\rho\delta_0$;
B, parameter, $B = (T_s - T_\infty)/\ln(P/P_\infty)$;
C, liquid specific heat;
e, constant, $e = 2.7182\ldots$;
F, dimensionless parameter, $F = \alpha/[1+(4/3)^{\frac{1}{2}}\beta\alpha^{\frac{1}{2}}\mathrm{d}I/\mathrm{d}m\,|_{\text{ave}}]$;
f, function of time;
I, indefinite integral, $I^2 = \int_0^\tau [\ln(p)]^2\,\mathrm{d}\tau$;
k, liquid thermal conductivity;
k_v, vapor thermal conductivity;
M, vapor mass per unit area in film;
m, dimensionless vapor mass per unit area, $m = M/\rho_v\delta_0$;
n, integer;
P, local pressure;
P_∞, pressure far from sphere;
p, dimensionless film pressure, $p = P/P_\infty$;
q_v, heat flow into interface from vapor;
q_L, heat flow from interface into liquid;
R, sphere radius;
r, radius;
T, temperature;
T_s, saturation temperature;
T_w, sphere temperature;
T_∞, liquid pool temperature;
t, time;
v, liquid radial velocity;
x, distance into liquid from interface, $x = r-(R+\delta)$.

Greek symbols

α, dimensionless parameter, $\alpha = [k_v(T_w - T_\infty)/\rho_v\Lambda](R\rho/\delta_0^3 P_\infty)^{\frac{1}{2}}$;
α_L, liquid thermal diffusivity;
β, dimensionless parameter, $\beta^2 = (k\rho/k_v\rho_v)[B/(T_w - T_\infty)][CB/\Lambda]$;
Δ, penetration depth of temperature disturbance into liquid;
δ, vapor film thickness;
δ_0, initial vapor film thickness;
θ, dimensionless liquid temperature, $\theta = (r/R)(T-T_\infty)/B$;
Λ, heat of vaporization;
γ, dimensionless parameter, $\gamma = (\delta_0/R)kB/[k_v(T_w - T_\infty)]$;
ρ, liquid density;
ρ_v, vapor density at ambient pressure and average temperature;
τ, dimensionless time, $\tau = At$.

Superscripts

$\cdot$, first ordinary time derivative;
$\cdot\cdot$, second ordinary time derivative.

INTRODUCTION

THE PRESSURE excursions which follow sudden exposure of a hot body to a liquid pool have been the subject of several recent studies with the safety of nuclear reactors as an important immediate application. Not only might nearby equipment and structures be affected by these pressure excursions, but it has also been conjectured that they could be responsible for the observed fragmentation of hot molten drops in contact with a liquid.

Experimental studies pertinent to this problem are few. Board *et al.* [1] measured pressure excursions arising from a metal foil suddenly heated by an electrical current while submerged in water and reported the pressure's amplitudes and frequencies. Flory, Paoli and Mesler [2] photographically studied the fragmentation of molten metal drops quenched in a liquid.

Most transient film boiling analyses assume pressure to be constant in the vapor film which lies between the hot body and the liquid and are not applicable to the subject problem. Rooney [3] accounted for the pressure excursions in the pure vapor film surrounding a constant temperature sphere immersed in a saturated liquid pool. He neglected heat conduction into the liquid and assumed the liquid to be incompressible. These simplifications permitted the oscillatory pressure excursions to be predicted at all times and for all ranges of the parameters considered by simple and analytically derived equations. However, his results give only the upper bound for the pressure excursion. Kazimi *et al.* [4] executed a detailed study of the same problem for a subcooled liquid, accounting not only for pressure excursions in the vapor film but also accounting for heat conduction in the liquid, liquid compressibility, finite heat capacity of the hot body, and the presence of some noncondensable gas in the vapor film. Because of the complexity of their mathematical model, numerical solutions specific to particular cases only were obtained. While they demonstrated that pressure excursions can be of appreciable magnitude and that the effects of liquid heat conduction are important, outweighing the effects of liquid compressibility, it is difficult to extend their results to other cases.

The purpose of the present work is to extend the analysis of Rooney to account for the effect of heat conduction in a saturated liquid while retaining sufficient simplicity to allow accurate analytical solution of the describing differential equations. The overall aim is to achieve solutions for the maximum pressure excursions which are easily applicable to a broad range of parameter values.

PROBLEM FORMULATION

As shown in Fig. 1, a sphere is immersed in a large pool of stagnant and saturated liquid with a thin film of vapor initially separating the sphere from the liquid. The sphere temperature is constant at a high value so that heat flows into the liquid–vapor interface by conduction across the vapor, generating additional vapor. Because the vapor is much less dense than the liquid, the liquid must ultimately be displaced away from the sphere which requires that the pressure in the film rise. In addition to accelerating the liquid away from the sphere, this pressure rise also increases the temperature of the liquid–vapor interface. As a result, heat is conducted into the liquid from the interface, leading to a vaporization rate which is diminished from its initial value. Because of the diminished vaporization rate and the resultant lessened need for vapor

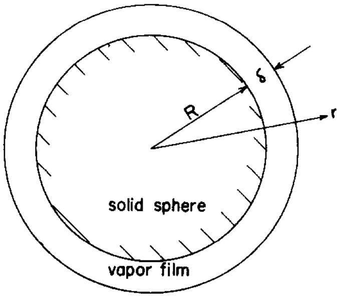

FIG. 1. Physical configuration and coordinate system.

volume, the liquid need not undergo as rapid a displacement and the film pressure rise is less than in the absence of heat conduction into the liquid.

Because of its inertia, the liquid later undergoes too large a displacement; the pressure in the film decreases below the ambient value, and the liquid then accelerates toward the sphere. The general result is that film thickness and pressure have an oscillatory behavior. The present study emphasises an examination of pressure behavior only up to the first pressure maximum which would be expected to be the largest one.

In the following analysis, a spherical geometry and a nonzero initial film thickness are assumed to avoid the infinitely large pressure excursions which otherwise are encountered for the assumed incompressible liquid. Gravitational body forces are neglected. The vapor film is taken to be thin enough for its curvature to be neglected and to always have a linear temperature distribution. It is also assumed that the liquid and vapor are in equilibrium at their interface at a temperature which corresponds to the instantaneous film pressure, and that the saturation temperature varies logarithmically with the film pressure.

The one-dimensional energy equation for the liquid is

$$\begin{aligned} \partial T/\partial t + v\partial T/\partial r &= \alpha_L(\partial^2 T/\partial r^2 + 2r^{-1}\partial T/\partial r) \\ T(0,r) &= T_\infty \\ T(t,R+\delta) &= T_s(P) \\ T(t,\infty) &= T_\infty \end{aligned} \tag{1}$$

which is recast into the form

$$\begin{aligned} \partial\theta/\partial t + (v-\dot{\delta})\partial\theta/\partial x - v(x+R+\delta)^{-1}\theta &\\ = \alpha_L\partial^2\theta/\partial x^2 &\\ \theta(0,x) = 0 &\\ \theta(t,0) = R^{-1}(R+\delta)\ln(P/P_\infty) &\\ \theta(t,\infty) = 0. & \end{aligned} \tag{2}$$

It has been determined by others that convection in the liquid is not important for the situation considered here [5]. Accordingly, equation (2) with convective terms neglected and with film thickness considered to be small relative to sphere radius simplifies to

$$\begin{aligned} \partial\theta/\partial\tau &= A^{-1}\alpha_L\partial^2\theta/\partial x^2 \\ \theta(0,x) &= 0 \\ \theta(\tau,0) &= \ln(p) \\ \theta(\tau,\infty) &= 0. \end{aligned} \tag{3}$$

An approximate, but accurate, solution to the simplified energy equation (3) is obtained by an integral method [6]. In integral form equation (3) is

$$\mathrm{d}\left(\int_0^\Delta \theta\,\mathrm{d}x\right)/\mathrm{d}\tau = -A^{-1}\alpha_L\partial\theta(\tau,0)/\partial x. \tag{4}$$

The spatial distribution of dimensionless temperatures is approximated by

$$\theta = (1-x/\Delta)^2\ln(p) \tag{5}$$

which is most accurate for monotonically increasing interfacial temperatures.

Introduction of equation (5) into equation (4) then gives

$$\mathrm{d}[\Delta\ln(p)]/\mathrm{d}\tau = 6A^{-1}\alpha_L\ln(p)/\Delta$$

from which it is found that

$$\Delta \ln(p) = (12A^{-1}\alpha_L)^{\frac{1}{2}} I \tag{6}$$

where

$$I^2 = \int_0^{\tau} [\ln(p)]^2 \, d\tau.$$

The heat flow into the liquid at the interface is evaluated from the relation

$$q_L = -k\partial T(\tau, R+\delta)/\partial r$$

and in conjunction with equation (6) to be

$$q_L = kR^{-1}B\ln(p) + B(4Ak\rho C/3)^{\frac{1}{2}} \, dI/d\tau. \tag{7}$$

Conservation of energy applied to the liquid–vapor interface requires that

$$\Lambda \, dM/dt = q_v - q_L. \tag{8}$$

Assuming that heat flows from the hot sphere to the interface by conduction through the thin vapor film, one has

$$q_v = k_v(T_w - T_s)/\delta \tag{9}$$

where it is assumed that $T_w - T_s$ is negligibly affected by variation of saturation temperature. The vapor is taken to be a perfect gas so that one also has

$$M = \delta\rho_v P/P_\infty. \tag{10}$$

Introducing equations (7), (9), and (10) into equation (8) then gives

$$dm/d\tau = \alpha[p/m - \gamma\ln(p)] - (4\beta^2\alpha/3)^{\frac{1}{2}} \, dI/d\tau. \tag{11}$$

The one-dimensional continuity and momentum equations in spherical coordinates for the incompressible liquid are

$$\partial(r^2 v)/\partial r = 0 \tag{12}$$

and

$$\partial v/\partial t + \tfrac{1}{2}\partial v^2/\partial r = -\rho^{-1}\,\partial P/\partial r. \tag{13}$$

Equation (12) shows that

$$r^2 v = f(t). \tag{14}$$

Substitution of this result into the momentum equation (13) followed by integration from $r = R+\delta$ to $r = \infty$ gives

$$\dot{f}/(R+\delta) - \tfrac{1}{2}f^2/(R+\delta)^4 = (P - P_\infty)/\rho. \tag{15}$$

Conservation of mass applied to the liquid–vapor interface requires that

$$\dot{M} = \rho(\dot{\delta} - v). \tag{16}$$

Substitution of equations (10) and (14) into equation (16) gives

$$f/(R+\delta)^2 = (P_\infty/\rho_v)\, d(M/P)\, d\tau - \dot{M}/\rho. \tag{17}$$

Taking the time derivative of equation (17) results in

$$\dot{f}/(R+\delta)^2 - 2f\dot{\delta}/(R+\delta)^3 = (P_\infty/\rho_v)\, d^2(M/P)/dt^2 - \ddot{M}/\rho. \tag{18}$$

Upon substitution of equations (10), (17), and (18) into equations (15) and putting into dimensionless form it is found that

$$\begin{aligned} &d^2(m/p)/d\tau^2 - (\rho_v/\rho)\, d^2m/d\tau^2 \\ &\quad + \tfrac{1}{2}(\delta_0/R)[d(m/p)/d\tau - (\rho_v/\rho)\, dm/d\tau] \\ &\quad \times [3d(m/p)/d\tau + (\rho_v/\rho)\, dm/d\tau] = p - 1. \end{aligned} \tag{19}$$

Because attention is focused on cases where the vapor film thickness is small relative to the sphere radius, terms for which δ_0/R is a coefficient can be deleted. Realizing, in addition, that the vapor is much less dense than the liquid ($\rho_v/\rho \ll 1$), equation (19) reduces to

$$d^2(m/p)/d\tau^2 = p - 1. \tag{20}$$

Initial conditions imposed are that

$$\delta(t = 0) = \delta_0 \tag{21}$$

$$P(t = 0) = P_\infty \tag{22}$$

$$v(t = 0, r) = 0. \tag{23}$$

From equations (21) and (22) the corresponding dimensionless initial conditions are

$$m(\tau = 0) = 1 = p(\tau = 0). \tag{24}$$

Equation (16) in conjunction with equation (23) requires initially that

$$\dot{M} = \rho\dot{\delta}$$

which, when taken together with equation (10) gives the dimensionless initial condition

$$d(m/p)/d\tau = (\rho_v/\rho)\, dm/d\tau.$$

Again because $\rho_v/\rho \ll 1$, this initial condition can be represented as

$$d(m/p)/d\tau = 0. \tag{25}$$

For convenience and clarity the equations to be solved and their initial conditions are brought together

$$dm/d\tau = \alpha p/m - (4/3)^{\frac{1}{2}}\beta\alpha^{\frac{1}{2}}\, dI/d\tau \tag{11}$$

$$d^2(m/p)/d\tau^2 = p - 1 \tag{20}$$

and, at $\tau = 0$

$$m(\tau = 0) = 1 = p(\tau = 0) \tag{24}$$

$$d(m/p)/d\tau = 0. \tag{25}$$

The dimensionless film thickness is, from equation (10),

$$\delta/\delta_0 = m/p. \tag{26}$$

SOLUTIONS

Equations (11), (20), (24), and (25) must be solved to determine the dimensionless film pressure (p), mass per unit area (m), and thickness (δ/δ_0) and are characterized by a nonlinearity which makes recourse to a numerical solution unavoidable in general. It must be remembered that the temperature profile used in the integral solution of the liquid's energy equation is most accurate for monotonically increasing interface temperatures. Accordingly, any solutions obtained are most accurate to the time at which film pressure attains its first maximum.

The two parameters, α and β, have opposite major influence as is seen by inspection of equation (11). The parameter α can be interpreted as the initial rate of dimensionless vaporization. Since vaporized mass requires a pressure excursion to displace liquid away from the sphere, large pressure excursions are expected to be associated with large values of α. And, small pressure excursions are expected when α is small.

The parameter β can be interpreted as the ratio of the conductive heat flux into the semi-infinite liquid pool (with a step change of interface temperature of magnitude B), evaluated after a time interval equal to that required for the initial vapor mass to be vaporized by conduction through the vapor at the initial rate, to the initial conductive heat flux through the vapor. Inasmuch as heat conduction into the liquid reduces the vaporization rate, β acts primarily to diminish the pressure excursion. Accordingly, large values of β are expected to be associated with small pressure excursions while small values of β are expected to be associated with pressure excursions only slightly below those predicted by Rooney for $\beta = 0$. As equation (11) shows, large values of α have a greater effect upon the heat conduction through the vapor than upon the heat conduction into the liquid. Accordingly, pressure excursions are expected to increase with increasing values of α even for large values of β.

Their definitions show that β is independent of initial film thickness while α is strongly dependent on it. Since the initial film thickness for a specific physical situation is somewhat uncertain, β is likely to be known with more certainty than is α.

Numerical method

The general case was solved by application of the MIMIC program on an H635 digital computer. The step size used in the numerical integrations was automatically adjusted to maintain a prescribed accuracy.

Small excursions

For small pressure excursions a simple approximate solution can be obtained. Equation (11) can be rewritten as

$$\mathrm{d}m/\mathrm{d}\tau = \alpha p/m-(4/3)^{\frac{1}{2}}\beta\alpha^{\frac{1}{2}}(\mathrm{d}I/\mathrm{d}m)(\mathrm{d}m/\mathrm{d}\tau)$$

and rearranged into the form

$$\mathrm{d}m/\mathrm{d}\tau = Fp/m \tag{27}$$

where $F = \alpha/[1+(4/3)^{\frac{1}{2}}\beta\alpha^{\frac{1}{2}}\,\mathrm{d}I/\mathrm{d}m|_{\text{ave}}]$ is assumed to be a constant which can be evaluated later, an insight gleaned from the numerical solutions.

Equations (20), (26), (24), and (25) are then of the same form as those solved by Rooney. The solution is

$$\ln(p) = Fm^{-\frac{1}{2}}\sin[(2/3F)(m^{\frac{3}{2}}-1)]. \tag{28}$$

From this solution it is found that

$$\ln(p_{\max}) \approx F$$

or

$$p_{\max} \approx 1+F \tag{29}$$

at

$$m_{\max} \approx (1+3\pi F/4)^{\frac{2}{3}} \tag{30}$$

and

$$\tau_{\max} \approx \pi/2. \tag{31}$$

I can be evaluated now by rearranging its definition into

$$I^2 = \int_1^m [\ln(p)]^2(\mathrm{d}\tau/\mathrm{d}m)\,\mathrm{d}m.$$

Incorporating equation (27) into this relation gives

$$FI^2 = \int_1^m [\ln(p)]^2(m/p)\,\mathrm{d}m.$$

For the case where $p \approx 1$, one then has

$$FI^2 \approx \int_1^m [\ln(p)]^2 m\,\mathrm{d}m.$$

Introducing equation (28) yields

$$I^2 = (F^2/4)(z-\sin z) \tag{32}$$

where

$$z = (4/3F)(m^{\frac{3}{2}}-1).$$

Because equation (32) gives $\mathrm{d}I/\mathrm{d}m$ as a nearly constant function, $\mathrm{d}I/\mathrm{d}m$ is approximated by

$$\mathrm{d}I/\mathrm{d}m|_{\text{ave}} = (m_{\max}-1)^{-1}\int_1^{m_{\max}} (\mathrm{d}I/\mathrm{d}m)\,\mathrm{d}m$$

from which, in conjunction with equation (32) it is found that

$$\mathrm{d}I/\mathrm{d}m|_{\text{ave}} = F(\pi/4)^{\frac{1}{2}}[(1+3\pi F/4)^{\frac{2}{3}}-1]^{-1}.$$

For the limiting case where pressure is expected to be small (small values of F)

$$\mathrm{d}I/\mathrm{d}m|_{\text{ave}} = \pi^{-\frac{1}{2}}.$$

This result substituted into equation (29) gives

$$p_{\max} = 1+\alpha/[1+(4/3\pi)^{\frac{1}{2}}\beta\alpha^{\frac{1}{2}}]. \tag{33}$$

With the understanding that m is but little affected by pressure excursions when F is small, equation (26) reveals that the ratio of film thickness with and without pressurization is

$$\delta_{\text{pressure}}/\delta_{\text{no pressure}} \approx 1 \pm F$$

Thus, if there is to be less than a 10% influence of pressure excursions upon film thickness it is necessary that

$$F = \alpha/[1+(4/3\pi)^{\frac{1}{2}}\beta\alpha^{\frac{1}{2}}] \leqslant 0.1. \tag{34}$$

The pressure oscillation's frequency and amplitude are given by the same expressions previously given by Rooney, but with F from equation (34) used in place of α.

Large excursions

The large pressure excursions that are expected when α is very large can also be predicted by an approximate solution. For this case, equation (20) is accurately approximated as

$$\mathrm{d}^2(m/p)/\mathrm{d}\tau^2 = p. \tag{35}$$

Repeating the argument advanced in connection with small pressure excursions, equation (11) is again approximated by equation (27).

Equations (35), (27), (24), and (25) are then of the same form as previously solved by Rooney. The solution is

$$p = m\,\mathrm{e}^{-m^3/6F^2} \tag{36}$$

from which it is found that

$$p_{\max} = (2F^2/e)^{\frac{1}{3}} \tag{37}$$

at

$$m_{\max} = (2F^2)^{\frac{1}{3}} \tag{38}$$

and

$$\tau_{\max} = (2/F)^{\frac{1}{3}} \tag{39}$$

showing that

$$\delta_{\max}/\delta_0 \approx e^{\frac{1}{3}}.$$

Again, it is necessary to evaluate I which was previously shown to be available from the relationship

$$FI^2 = \int_1^m [\ln(p)]^2 (m/p)\,\mathrm{d}m.$$

Introducing equation (36) into this gives

$$FI^2 = \int_1^m [\ln(m) - m^3/6F^2]^2\,\mathrm{e}^{m^3/6F^2}\,\mathrm{d}m.$$

Expanding the squared term in the integrand and integrating by parts leads to

$$\begin{aligned} FI^2 = {} & \int_1^m [\ln(m) + 1/3]^2\,\mathrm{e}^{m^3/6F^2}\,\mathrm{d}m + \int_1^m \mathrm{e}^{m^3/6F^2}\,\mathrm{d}m \\ & - (2/3)m\ln(m)\,\mathrm{e}^{m^3/6F^2} + (m^4/18F^2)\,\mathrm{e}^{m^3/6F^2} \\ & - (4m/9)\,\mathrm{e}^{m^3/6F^2} - \mathrm{e}^{\frac{1}{6}F^2}/18F^2 + 4/9\,\mathrm{e}^{\frac{1}{6}F^2}. \end{aligned} \tag{40}$$

Anticipating again that $\mathrm{d}I/\mathrm{d}m$ does not vary much with m, an average value is obtained as

$$\mathrm{d}I/\mathrm{d}m_{\mathrm{ave}} = (m_{\max} - 1)^{-1} \int_1^{m_{\max}} (\mathrm{d}I/\mathrm{d}m)\,\mathrm{d}m$$

or,

$$\mathrm{d}I/\mathrm{d}m_{\mathrm{ave}} = (m_{\max} - 1)^{-1} I(m_{\max})$$

where $m_{\max}$ is given by equation (38). The integrals in equation (40) are evaluated by expanding $\mathrm{e}^{m^3/6F^2}$ in series according to

$$\mathrm{e}^{m^3/6F^2} = \sum_{n=0}^{\infty} (m^3/6F^2)^n/n!$$

which ultimately yields

$$\begin{aligned} \mathrm{d}I/\mathrm{d}m|_{\mathrm{ave}} = {} & 0.8192(2F^2/e)^{\frac{5}{12}}[\{\ln(2F^2/e)^{\frac{1}{3}} - 0.6954\}^2 \\ & + 0.8443 - 0.6562(2F^2/e)^{-\frac{1}{3}}]^{\frac{1}{2}}. \end{aligned}$$

Because of the relationship between $p_{\max}$ and F given by equation (37), this result can be put into the more convenient form

$$\begin{aligned} \mathrm{d}I/\mathrm{d}m|_{\mathrm{ave}} = {} & 0.8192 p_{\max}^{-\frac{5}{4}}[\{\ln(p_{\max}) - 0.6954\}^2 + 0.8443 \\ & - 0.6562/p_{\max}]^{\frac{1}{2}}. \end{aligned} \tag{41}$$

Recalling again the relation between $p_{\max}$ and F given by equation (37) and the definition of F, the final result is

$$\begin{aligned} (e/2)^{\frac{1}{2}} p_{\max}^{\frac{3}{2}} \{1 + 0.946 p_{\max}^{-\frac{5}{4}}[\{\ln(p_{\max}) - 0.6954\}^2 \\ + 0.8443 - 0.6562/p_{\max}]^{\frac{1}{2}} \beta \alpha^{\frac{1}{2}}\} = \alpha. \end{aligned} \tag{42}$$

Although equation (42) is an implicit relation for $p_{\max}$ when α and β are known, it is still useful because of its accuracy and algebraic nature. A practical procedure for its use is to first solve it for $\alpha^{\frac{1}{2}}$ in terms of $p_{\max}$ and β. Then successive assumed values of $p_{\max}$ are used with the known value of β until the calculated α equals the known value of α. More accuracy can be obtained if the B and $T_w - T_s$ values used to evaluate α and β are determined at an average pressure in an iterative manner.

DISCUSSION

The results of the calculations for maximum film pressure are shown in Fig. 2. There it is seen that the approximate solutions for large and small values of the parameter α are in good agreement with the numerical solutions except for the $\beta = 0$ case, where the approximate solutions are inaccurate at intermediate values of α.

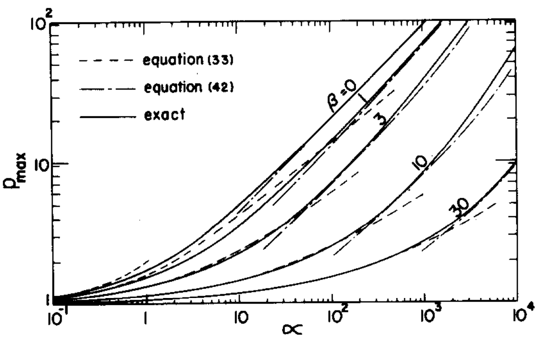

FIG. 2. Dimensionless maximum pressure as a function of the dimensionless parameters α and β.

Even though the factors of major influence are believed to have been accounted for in this study, the results for maximum pressure must still be regarded as upper bounds. Accounting for the compressibility of the liquid which is neglected here, for example, was shown by Kazimi *et al.*'s numerical calculations to lead to a slightly lower maximum pressure. The drop in surface temperature of the hot sphere caused by its loss of heat has also been neglected and would be expected to lead to a slightly lower maximum pressure. However, the generality of the present results is believed to justify the sacrifice in accuracy caused by neglecting factors of only secondary influence.

To illustrate the application of these results, consider a 500°C sphere of 0.3 cm radius suddenly put in contact with a 100°C pool of saturated water. For these conditions, $\beta = 13$.

If the initial vapor film thickness is 3×10^{-3} cm, $\alpha = 0.6$ and $A = 3.5 \times 10^4 \text{ s}^{-1}$. Figure 2 and equation (33) show that the maximum pressure is $1.08 \times 10^5 \text{N/m}^2$, representing a pressure excursion of $0.08 \times 10^5 \text{N/m}^2$ and occurring after 45×10^{-6} s according to equation (31). The importance of heat conduction in the liquid, embodied in the parameter β, is shown by the overprediction that the maximum pressure excursion is $0.5 \times 10^5 \text{N/m}^2$ if $\beta = 0$. The qualitative agreement of this case with the experimental pressure measurements of Board is good inasmuch as he reported pressure excursions of about 6900 N/m^2. Also, equation (34) shows that this pressure excursion would have less than a 10% influence on the growth of the film.

At the other extreme, if the initial vapor film thickness is 10^{-5} cm, $\alpha = 3120$ and $A = 61 \times 10^4 \text{ s}^{-1}$. Figure 2 and equation (42) indicate that the maximum pressure is $12.5 \times 10^5 \text{N/m}^2$, representing a pressure excursion of $11.5 \times 10^5 \text{N/m}^2$ and occurring after 0.55×10^{-6} s according to equation (39). If heat conduction in the liquid had been ignored by setting $\beta = 0$, a pressure excursion of $192 \times 10^5 \text{N/m}^2$ would be predicted which testifies again to the importance of liquid heat conduction. Although Kazimi *et al.*, only executed calculations for a subcooled liquid with some noncondensable gas initially present in the film, at their lowest subcooling of 20°C they obtained a maximum pressure of $6.8 \times 10^5 \text{N/m}^2$ after 0.9×10^{-6} s with an incompressible liquid for the conditions of this example.

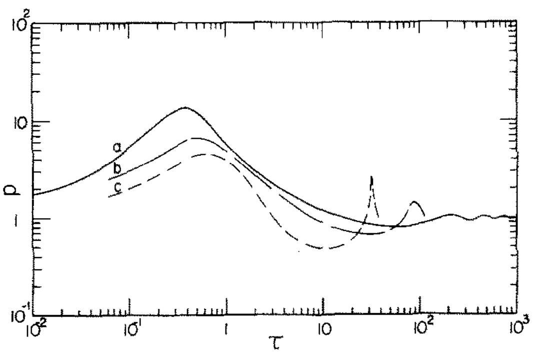

FIG. 3. Dimensionless pressure as a function of dimensionless time for $\alpha = 3120$ and $\beta = 13$: (a) present study with saturated liquid; Kazimi *et al.* [4] for water with ambient pressure of 1 atm at (b) 20°C subcooling and (c) 50°C subcooling.

This is only fair agreement, although the difference is in the proper direction since subcooling should yield a lower pressure. A plausible relation between the pressure excursions for saturated and slightly subcooled liquid pools is seen in Fig. 3 for the case where $\alpha = 3120$ and $\beta = 13$.

The initial film thickness, as mentioned before, is likely to be an uncertain quantity whose magnitude depends upon the specific circumstances encountered. Figure 2 and the foregoing illustrative examples indicate that if β is large, the maximum pressure excursion will be small and does not vary much with α (which is dependent upon initial film thickness) so that the maximum pressure is rather insensitive to the initial conditions. Liquid compressibility can be safely neglected under these conditions. At the other extreme where β is small, the maximum pressure excursion is quite sensitive to the initial conditions and liquid compressibility would be of some importance.

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INFLUENCE DE LA CONDUCTION THERMIQUE DU LIQUIDE SUR LE MAXIMUM DE PRESSION POUR L'EBULLITION EN FILM AUTOUR D'UNE SPHERE D'UN LIQUIDE SATURE

Résumé—On évalue les pointes de pression maximale qui se produisent dans le film de vapeur pure qui enveloppe une sphère très chaude et à température constante, mise en contact brusque avec un liquide saturé au repos dans un réservoir. On montre que la conduction thermique dans le liquide incompressible, liée à la dépendance de la température de saturation vis à vis de la pression, est très importante. On présente, sous forme de graphes et d'équations approchées, les prévisions généralisées de la pression maximale.

DER EINFLUSS DER FLÜSSIGKEITSWÄRMELEITUNG AUF DEN BEIM INSTABILEN FILMSIEDEN EINER GESÄTTIGTEN FLÜSSIGKEIT AN EINER KUGEL AUFTRETENDEN MAXIMALDRUCK

Zusammenfassung—Beim plötzlichen Eintauchen einer sehr heißen Kugel konstanter Temperatur in eine ruhende, gesättigte Flüssigkeit entsteht ein reiner Dampffilm um die Kugel. Die dabei auftretenden maximalen Drücke werden analytisch ausgewertet. Infolge der Druckabhängigkeit der Sättigungstemperatur ist die Wärmeleitung in der inkompressiblen Flüssigkeit von besonderer Bedeutung. Der Maximaldruck wird in Form verallgemeinerter Diagramme wiedergegeben; eine Näherungsgleichung wird abgeleitet.

ВЛИЯНИЕ ТЕПЛОПРОВОДНОСТИ ЖИДКОСТИ НА МАКСИМАЛЬНОЕ ДАВЛЕНИЕ ПРИ НЕСТАЦИОНАРНОМ ПЛЕНОЧНОМ КИПЕНИИ НА ПОВЕРХНОСТИ ШАРА, ПОГРУЖЕННОГО В НАСЫЩЕННУЮ ЖИДКОСТЬ

Аннотация — Дан аналитический расчет максимальных отклонений давления в пленке чистого пара, покрывающей сильно нагретый шар с постоянной температурой, при его внезапном погружении в объем насыщенной и неподвижной жидкости. Показана важность процесса теплопроводности в несжимаемой жидкости, возникающего вследствие зависимости температуры насыщения от давления. Графически представлены обобщенные зависимости для максимального давления и выведены приближенные уравнения.